(3) $\dfrac{(5-3i)(2+i)}{4+2i} = \dfrac{10+5i-6i-3i^2}{4+2i} = \dfrac{10+3-i}{4+2i} = \dfrac{13-i}{4+2i} \times \dfrac{4-2i}{4-2i}$

$= \dfrac{52-4i-26i+2i^2}{4^2-(2i)^2} = \dfrac{52-30i-2}{16-4i^2}$

$= \dfrac{50-30i}{16+4} = \dfrac{50-30i}{20} = \dfrac{5}{2} - \dfrac{3i}{2} = a+ib$

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(4) ~~1+2i~~ $\dfrac{(1+i)(2+i)}{3+i} = \dfrac{2+i^2+2i+i}{3+i} = \dfrac{2-1+3i}{3+i}$

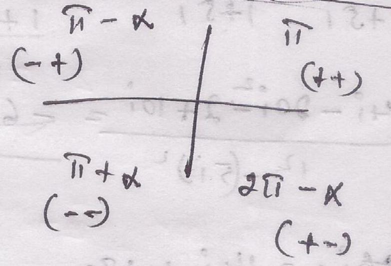
$= \dfrac{1+3i}{3+i} \times \dfrac{3-i}{3-i} = \dfrac{(1+3i)(3-i)}{3^2-i^2}$

$= \dfrac{3-i+9i-3i^2}{9+1} = \dfrac{3+8i+3}{10}$

$= \dfrac{6+8i}{10} = \dfrac{3+4i}{5} = 3/5 + 4i/5$

Cube root

$r = ?$
$\alpha = ?$
quadrant = ?
$\theta = ?$



(1) $1-i$

$\Rightarrow$ Let $z = x+iy = 1-i$

$\Rightarrow r = \sqrt{x^2+y^2} = \sqrt{1+1} = \sqrt{2}$

$\alpha = \tan^{-1}\left|\dfrac{y}{x}\right| = \tan^{-1}\left|\dfrac{-1}{1}\right| = \pi/4$ radian

Here $z = 1-i = (1,-1)$ is in 4th quadrant

$\theta = 2\pi - \alpha = 2\pi - \pi/4 = \dfrac{7\pi}{4}$

therefore, $z = \sqrt{2}\left[\cos\left(\dfrac{7\pi}{4}\right) + i\sin\left(\dfrac{7\pi}{4}\right)\right]$

or $z = \sqrt{2}\left[\cos\left(\dfrac{2k\pi+7\pi}{4}\right) + i\sin\left(\dfrac{2k\pi+7\pi}{4}\right)\right]$

$$= 2^{1/2}\left[\cos\left[\frac{8k+7}{4}\right]\pi + i\sin\left[\frac{8k+7}{4}\right]\pi\right]$$

$$= z^{1/3} = 2^{1/6}\left[\cos\left[\frac{8k+7}{4}\right]\pi + i\sin\left[\frac{8k+7}{4}\right]\pi\right]^{1/3}$$

$$z_k = z^{1/3} = 2^{1/6}\left[\cos\left[\frac{8k+7}{12}\right]\pi + i\sin\left[\frac{8k+7}{12}\right]\pi\right] \quad \text{where } k = 0,1,2.$$

$$z_0 = 2^{1/6}\left[\cos\frac{7\pi}{12} + i\sin\left[\frac{7\pi}{12}\right]\right]$$

$$z_1 = 2^{1/6}\left[\cos\frac{15\pi}{12} + i\sin\frac{15\pi}{12}\right]$$

$$z_2 = 2^{1/6}\left[\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right]$$

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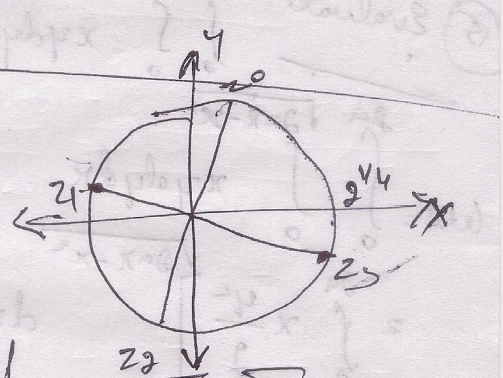
(2) $1 - i\sqrt{3}$ $\boxed{4^{th} \text{ roots}}$



let $z = 1 - i\sqrt{3} = x + iy$

$$\boxed{r} = \sqrt{x^2 + y^2} = \sqrt{1+3} = 2$$

$$\boxed{\alpha} = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{-\sqrt{3}}{1}\right| = \tan^{-1}\sqrt{3} = \frac{\pi}{3}.$$

Here $z = 1 - i\sqrt{3} = (1, -\sqrt{3})$ is in 4th quadrant.

$(+,-)$ $\boxed{\theta} = 2\pi - \alpha = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

therefore, $z = 2\left[\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right]$

or $z = 2\left[\cos\left(2k\pi + \frac{5\pi}{3}\right) + i\sin\left(2k\pi + \frac{5\pi}{3}\right)\right]$

$$z = 2\left[\cos\left(\frac{6k+5}{3}\right)\pi + i\sin\left(\frac{6k+5}{3}\right)\pi\right]$$

therefore, $z_k = z^{1/4} = 2^{1/4}\left[\cos\left[\frac{6k+5}{3}\right]\pi + i\sin\left[\frac{6k+5}{3}\right]\pi\right]^{1/4}$ 4th root

$$z_k = 2^{1/4}\left[\cos\left[\frac{6k+5}{12}\right]\pi + i\sin\left[\frac{6k+5}{12}\right]\pi\right] \quad \text{where } k = 0,1,2,3.$$

$$z_0 = 2^{1/4}\left[\cos \frac{5\pi}{12} + i\sin\left(\frac{5\pi}{12}\right)\right]$$

$$z_2 = 2^{1/4}\left[\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right]$$

$$z_1 = 2^{1/4}\left[\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right]$$

$$z_3 = 2^{1/4}\left[\cos 23\pi/12 + i\sin 23\pi/12\right]$$